

Language of Mathematics Task Templates

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PREFACE

The mathematics tasks with annotations (available on the UL web site) provide examples of how teachers can use a mathematics task that is aligned with the CCSS when working with ELLs. Each annotation includes at least one Language of Mathematics task designed to support students in learning to read and understand word problems, or review vocabulary, or communicate about a mathematical problem they have solved. In this Appendix, we provide templates for five of these Language of Mathematics tasks.

Teachers can use these five templates to design and write their own Language of Mathematics tasks to fit a mathematics task of their choice.

Student materials and descriptions for the Language of Mathematics Task Templates were either adapted from the Understanding Language unit in English Language Arts (Walqui, Kolesch, & Schmida, 2012), adapted from materials used by R. Santa Cruz and H. Asturias, both professionals in teacher professional development, or recommended as “good bets” by researchers with expertise in how to scaffold vocabulary and reading comprehension. The materials draw, in large part, on papers prepared for the Understanding Language Conference at Stanford University (<http://ell.stanford.edu/papers>) and the ELA unit written by Walqui, Kolesch, & Schmida (http://ell.stanford.edu/teaching_resources/ela).

These templates for the Language of Mathematics tasks focus on two issues: reading word problems and using vocabulary to communicate about solutions to a mathematics problem. This is not because we think that these are the most central for learning mathematics *or* language but, instead, for several pragmatic reasons. First, teachers have often raised these two issues as the major stumbling blocks they face when teaching ELLs mathematics. Second, there were existing exemplars of activities addressing these two issues that had already been piloted with students and teachers and used in professional development work by R. Santa Cruz and H. Asturias.

In this Appendix, we provide templates for each task type with a general description for how to organize each activity. However, the best way to see how these Language of Mathematics tasks work when used with a CCSS-aligned mathematics task is to look at the following mathematics tasks with annotations on the Understanding Language website:

Elementary School | *Roger's Rabbits*

Mathematically Speaking (p.11-14)

Middle School | *Making Matchsticks*

Mathematically Speaking (p. 12-15)

High School | *Sidewalk Patterns*

Reading and Understanding a Mathematics Problem

(p.14-19)

High School | *Creating Equations*

Jigsaw Reading (p. 14-29)

Reading and Understanding a Mathematics Problem

(p. 21-24)

Mathematically Speaking (p. 25-29)

TASKS TO SUPPORT READING MATHEMATICS PROBLEMS

We provide two templates for Language of Mathematics tasks to support students in learning to read and understand word problems:

1. Reading and Understanding a Mathematics Problem (*page 37*)
2. Jigsaw Reading (*page 41*)

READING AND UNDERSTANDING A MATHEMATICS PROBLEM

Adapted from handout developed by Harold Asturias.

Purpose

The purpose of Reading and Understanding a Mathematics Problem is to support students in learning to approach a mathematics problem. It gives students tools for learning to read, understand, and extract relevant information from a problem. It also gives students practice in identifying additional information they need in order to solve the problem.

Required for use

- Handout: Student materials.
- A mathematics task using realistic quantities or a real world scenario (not necessarily actual data) that requires reading text. The task should provide some information while omitting other information necessary to solve the problem.

Structure of the activity

1. Students begin by reading or attempting to read the problem individually.
2. Students then form into pairs and work together to talk through the problem using the handout provided. There are five steps in talking through the problem together, three of which begin with reading the problem aloud.

Step 1 Students identify what the problem is about (marbles; concert tickets; or a rectangle).

Step 2 Students are asked to make explicit what information they need to find (a number of marbles; the numbers of two kinds of tickets, each with a different price; or the length and width of a rectangle).

Step 3 Students answer these questions together, both orally and in writing.

Step 4 Teacher asks a scaffolded set of questions leading to a diagram that represents both the known and unknown information about the quantities in the situation.

Step 5 Teacher asks students to try to act out the problem using real objects to represent the quantities in the situation.

3. Pairs of students should present their diagrams to the class. As they view and interpret other students' diagrams, they should add details or labels to their own.

Process outline

1. Students work individually on the problem.
2. Students form pairs and each pair shares one copy of the handout.
3. Pairs of students talk together to answer the questions in Steps 1–3 on the handout in writing.
4. Finally students try to act out the problem using physical objects to represent the quantities in the situation.

Name _____

Date _____

STUDENT MATERIALS: READING AND UNDERSTANDING A MATHEMATICS PROBLEM

Step 1. Read the problem out loud to a peer. Try to answer this question.

What's the problem about?

Step 2. Read the problem again. Talk to your partner about these questions:

What is the question in the problem?

What are you looking for? (Hint: Look at the end of the problem for question.)

Adapted from handout developed by Harold Asturias

Step 3. Read the problem a third time. Talk to your partner about these questions.

- a) What information do you need to solve the problem? (What do you want to know?)
- b) What information do you have? (What do you know?)
- c) What information are you missing? (What don't you know?)
- d) Draw a diagram of the problem and label all the information you know.

Information

Step 4. (If useful for this problem) Draw a diagram, act the problem out, use objects to represent the problem situation.

Adapted from handout developed by Harold Asturias

JIGSAW READING

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).

Note: Jigsaw Reading is an activity in a very early draft form and has not yet been piloted in classrooms. It may turn out to work best with longer complex mathematics text.

Purpose

This task aims to alert students to the organization of a math problem or text and the discourse and content connections that make texts flow and be predictable. For example, the structure of a math problem begins with the statement of some given information, then there may be more information, and finally there is a question or request for a solution or missing information. In this activity, students are given one piece of a problem, and they must read closely to determine where in a math problem their section fits. In the process, students begin to focus, without prompting, on how grammatical and lexical choices create cohesion and meaning within and across sentences and how larger units of text are connected to create coherence or a unity of meaning. The activity apprentices students into the type of close reading needed to understand more complex math problems and math texts.

Required for use

An ideal math problem or text for this treatment should be no longer than a half page. Initially, the sections should contain clear markers of organization that are characteristic for that type of problem. As students become more sophisticated readers of math problems they may benefit from reading and reassembling texts that are clearly organized but do not use “set” markers to signal organization.

Structure of the activity

Initially, the teacher sets out the overall purpose of the task by explaining that writers of math problems use language to connect ideas within and across paragraphs in a text. The teacher should explain that students will be given sections of a math problem that has been divided into three pieces. They will reassemble the problem by putting the pieces in order, and this will help them understand how math problems work. The teacher might introduce the task with a math problem or text that is familiar to the class.

The selected text is cut into its sections that are placed in an envelope (the number of sections determines the number of students in a group). The teacher distributes and review the directions.

Process outline

4. One student distributes the text sections randomly to the group members.
5. Each student then reads his or her section silently and tries to imagine where the segment fits into a whole: Is it a beginning? The middle? The end? What makes them think so? Students must have reasons for their thinking.
6. When everyone in a group appears to be ready, the person who thinks he or she has the first piece says, “I think I have the first piece because...” and without reading the text aloud explains what clues led to this supposition. If any other group members think they have the first piece, then they too must explain, “I think I have the first piece because...” Once the group decides what piece should go first, the person with that piece reads it aloud.
7. After hearing the piece read aloud, the group discusses whether it is indeed the first piece. If agreement is reached, the piece goes face up on the table where group members can refer to it as needed.
8. Students follow the same procedure to reconstruct the rest of the text, section by section.
9. If students feel they have made a mistake along the way, they may go back and correct it.
10. Once the whole process is finished, all group members review the jigsawed text to make sure it has been assembled correctly.
11. The teacher can facilitate a whole group discussion asking students to explain which connectors or other linguistic features helped them to ascertain the order of the sentences. The teacher can use strips of transparencies on an overhead projector or paper strips on a white board to manipulate during student explanations. Where warranted, the teacher can provide alternative ways of stating similar relationships to those from the math problem.

TASKS TO SUPPORT VOCABULARY FOR MATHEMATICAL COMMUNICATION

We provide three templates for Language of Mathematics tasks to support students in learning to use vocabulary to communicate about their solutions to a mathematics problem.

1. Vocabulary Review Jigsaw (*page 44*)
2. Mathematically Speaking (*page 47*)
3. Vocabulary Pieces, Roots, and Families (*page 50*)

Overall, there are several ways to work with vocabulary, including introducing, using and reviewing terms. It is important to note that vocabulary need not be pre-taught or introduced in isolation but instead it should be included in activities that involve cognitively-demanding work—for example, reasoning, sense making, explaining and comparing solutions to a mathematics problem. Thus, decisions to “pre-teach” vocabulary should be carefully considered and should include activities where students *actively* solve and discuss their solutions to a mathematics problem.

When introducing new vocabulary, it is useful to first give students the opportunity for an engaging and successful problem-solving experience. Having grounded meanings by first solving a mathematics problem and discussing their solutions, the teacher can then label, define, and review technical terms.

Importantly, then, the purpose of vocabulary work should not be *primarily* for students to learn English vocabulary but to provide access to the mathematical work. The purpose of vocabulary work should be to provide opportunities for students to use mathematical language to communicate about how they solved a problem, describe their reasoning, explain why a solution or step works, and/or justify a claim, etc. Students will then learn English vocabulary as they engage in solving and discussing substantive mathematics problems. Lastly, tasks and activities that engage students in learning vocabulary should address several types of terms.

In this example: *Jane, Maria, and Ben each has a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.*

- Contextual or colloquial terms and phrases, for example, *marble*.
- Vocabulary specific to mathematics, for example, *more marbles than* or *2 times as many marbles as*.
- General academic language, for example, *find, describe, analyze*.

VOCABULARY REVIEW JIGSAW

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).

Note: Vocabulary Review Jigsaw is an activity in a very early draft form. The activity has not yet been piloted in mathematics classrooms.

Purpose

This task gives students an opportunity to review vocabulary or terms. Students work in groups of four to combine the clues held by each member and try to guess the 12 target words. It is important to recognize that this task is intended to be used not to teach vocabulary but to *review* vocabulary.

Required for use

The teacher selects 12 key vocabulary items or terms that the students have been introduced to within a task, lesson, or unit. The teacher prepares five cards—four to be used in the jigsaw, and one for the answer key.

There are two ways to prepare the jigsaw cards (Version I and Version II). The teacher provides four clues that will help student identify each vocabulary word. In Version I, the clues for each word fall into four categories. Three of the categories are very simple: (a) the first letter, (b) the number of syllables, and (c) the last letter. The fourth clue, (d), is a working definition of the term. The definition is not one from the dictionary; rather, the definition should be written by the teacher, using knowledge stressed in class.

In Version II, all the clues are meaningful. Clue A should be the broadest, opening up many possibilities. Clue B, while narrowing the selection of an answer, should still leave it quite open. Clue C should narrow the possibilities further. Clue D should limit the possibilities to the target word.

Structure of the activity

Students need to be in groups of four. The teacher explains to students that they will participate in a fun way to review vocabulary. Then the teacher models the activity. It should be stressed to students that the activity is collaborative and that all four clues (A, B, C, and D) must be heard before the group can guess the vocabulary word. The teacher should prepare a small jigsaw to use as an example, modeling the process with a key term the students have learned in a previous unit. For example, the teacher might choose the term “hyperbola” and prepare four index cards with the following clues:

- A: The first letter is “h.”
- B: There are four syllables.
- C: The last letter is “a.”
- D: The word means _____ .

The sample cards show two examples for “axis” and “intercept.”

Four students should work together to model for the class, with each student reading only his or her assigned clue.

CARD A

1. The word starts with the letter A.
2. The word starts with the letter I.

CARD C

1. The last letter in this word is x.
2. The last letter in this word is t.

CARD B

1. This word has TWO syllables.
2. This word has THREE syllables.

CARD D

1. It means the vertical and horizontal lines that determine the quadrants of a coordinate plane.
2. It means the point at which a line, curve, or surface intersects an axis.

Student’s Answer Sheet (sample)

1. _____
2. _____

Process outline

1. Students sit in groups of four.
2. Students number a piece of paper 1 to 12, down the left hand side.
3. The student with card A selects the number he or she would like to read, and all group members then circle the number on their answer sheets.
4. Each student reads their clue for that number, in order: A, B, C, and D.
5. After all four clues have been read, the students try to guess the word or term.
6. Students write their answer in the appropriate line on their answer sheet.
7. After three turns, students pass their cards to the person on their right so that all four students have a chance to read all four clue cards.
8. When a group has completed the jigsaw, one member asks for the answer key, and the group checks its answers, taking note of any terms that require additional study.

MATHEMATICALLY SPEAKING

Adapted from the work of R. Santa Cruz for the Understanding Language Initiative.

Purpose

This task gives students the opportunity to solve a problem and then explain and discuss how they arrived at their solution using targeted vocabulary. The activity is used for vocabulary review or guided practice.

The purpose of this task is to alert students to important vocabulary and terms *during* contextualized mathematics activity. Students are asked to listen for, track, and describe vocabulary they used while their group is solving a mathematics problem. It is crucial that students do this vocabulary work *after* they solve a mathematics problem that grounds the meanings for words.

Students will use everyday words while solving a mathematics problem or in early rounds of talking about their solutions with other students or the teacher, and they should not be corrected. Instead the teacher can provide more formal mathematical terms later during a whole class discussion.

Note that developing academic language involves more than just learning the target or specialized vocabulary of a unit or chapter. Comparative structures such as “twice as many, 3 less than 7” are syntactic structures that students also need as they use the target vocabulary of mathematics tasks.

Required for use

The teacher selects a set of key terms that the students have been introduced to within a task, lesson, or unit. The teacher prepares a chart or organizer for students to use.

Structure of the activity

All students independently complete both mathematical tasks or problems. They may use more space on a separate piece of paper, but must show their work

Students form pairs and each pair should get one copy of the Mathematically Speaking chart.

Target vocabulary words are written on the chart in the left column. For lower grades, the teacher can fill in the words. The two students write their names across the top. One student explains how they solved the

mathematics problem to the other student as the other student uses a checkmark on the chart to record each time a target word is used in the explanation. The other student then takes a turn doing the same.

Students can keep talking until all target words have been used.

This activity may be used for practice or assessment after students have worked on a mathematics problem and teachers have provided instruction or modeled the formal usage of the target vocabulary.

To support students in refining their descriptions and explanations, students can ask each other these questions:

- Did my explanation make sense?
- Do you have any questions about what I did?
- Do you have any questions about why I did this?

To focus on their mathematical reasoning, students can ask each other these questions:

- What did you do to solve the problem or find an answer?
- Why did you do that step?
- Why is that step justified mathematically?" or "What is a mathematical reason for that step?"

Process outline

1. Student pairs are formed.
2. Target vocabulary words are written on the Mathematically Speaking chart in the left column.
3. For lower grades, the teacher or volunteer may fill in the words.
4. The two students write their names across the top.
5. One student explains their solution to the other student as he or she writes a check on the chart each time a target word is used in the explanation.
6. Students keep talking until all target words have been used.
7. Students keep talking until all target words have been used.

STUDENT MATERIALS: MATHEMATICALLY SPEAKING

Date _____

Partner Names _____ & _____

Task Name _____

1. Solve the problem. Show your work
2. Explain your thinking, strategies, and solution to your partner. Use the target words in your explanation.
3. Listen to your partner's explanation and make a tally for each time he or she used the target vocabulary.

Explain how you solved the problem.

Explain how you solved the problem.

Problem 1	Problem 2
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Vocabulary Words Tally: How many times used

Vocabulary Words Tally how many times used

Example: variable	III
volume	
length	
greater than	
less than	
equal to	
units	

Example: Constant	IIII
volume	
units	
added	
combined	
array	

VOCABULARY PIECES, ROOTS, AND FAMILIES

Adapted from activities suggested by Judith Scott.

Note: This activity has not yet been piloted in mathematics classrooms.

Task One | Looking at word pieces in “equation”

Introduce the concept of a morpheme: Words made up of smaller pieces of meaning.

Ask students:

- Do they know what two parts make up the word *equation*? *equate + ion*.
- Do they know what the suffix *-ion* does? (This suffix turns the verb form of a word into a noun form.)
- The teacher asks students to "mine their brains" to come up with some other examples of words where *ion* turns a verb into a noun.
- The teacher records this on chart paper.
- The teacher asks students to collect examples of *-ion* words throughout the next 24 hours and to add examples to the chart.

For example: define, definition;

organize, organization;

explore, exploration;

equate, equation.

This is a highly productive pattern to learn in English.

Task Two | Looking at the root word *equate*.

- The teacher asks students what words they know that might be related in meaning to the root word for equation. Students can use WordSift.com or visualthesaurus.com to come up with ideas.
- The teacher asks students to collect words that they identify as related. For example: *equal, equivalent, equalize*.
- The teacher points out that all these words come from the Latin *aequare* which means to "make even or uniform, make equal" (p. 18).

Note that usage differs. Sometimes equations are considered to be expressions and sometimes they are not. In the Common Core State Standards, equations are not considered to be expressions.

Note also that the two expressions in an equation may not always be equal to each other. For example, an equation may be true for some values of its variables and false for others. Or an equation may simply be false, e.g., $0 = 1$.

- The teacher asks students, working in groups, or pairs, to pick two words from the list of identified words. Writing on sentence strips, students should explain how the meaning of each word relates to the Latin root.
- The teacher provides some example of good explanations:

Equation: An equation is a statement that two expressions are equal to each other.

Equivalent: If two algebraic expressions are equivalent, it means that they always have the same value.

Equivalent: If two numerical expressions are equivalent, it means that they have the same value. That is, they are equal to the same number (or both undefined).

Students may use everyday language to explain meanings of words in a way that makes sense to them. The teacher should have different groups read out the explanations until all the words are used.

Task Three | Finding Words in a Family

- Ask students to try to collect other words in the “equal” family.
- Post a chart that can be added to, or have students keep a record of these words in a “word catcher” notebook.

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